PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
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Phil. Trans. R. Soc. Lond. A 1997 **355**, 2387-2391
doi: 10.1098/rsta.1997.0135

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The double lambda system: a new workhorse for quantum optics?

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We review the status of the double-lambda system of cavity QED and mention unexpected recent applications in which deterministic and unitary control of quantum states is exploited.

The double-lambda system provides an attractive stripped-down quantum framework for modeling a variety of optical processes not compatible with the better-known Jaynes–Cummings (Jaynes & Cummings 1963) model. The main difference is the existence of two independent channels for exciting the atom in the Λ–Λ model. We are concerned here with the three- and four-mode Λ–Λ models, which are appropriate to the situations shown in figure 1, in which an atom or ion either slowly drifts through a cavity or is permanently trapped in a cavity. The figures show three radiation modes as if they were light beams incident from outside the cavity. We will consider situations in which at least one mode is resonant in the cavity.

The history of the double-lambda system is relatively recent. A list of contributions to quantum optics in which the double lambda, or close relatives of it, have played an important role would include discussions of amplification without inversion (Kocharovskaya 1990), near-resonant excitation via a classical and quantum channel (Law & Eberly 1991), two mechanisms for inversionless amplification (Keitel *et al.* 1993), two-mode squeezing with phase correlation (Law & Eberly 1993), proposal for GHZ state generation (Wodkiewicz *et al.* 1993), LWI (Fleischhauer *et al.* 1994), transparency and dressed fields in pair-photon propagation (Cerboneschi & Arimondo 1995), a photon 'engine' for generating an arbitrary single-mode photon state (Law & Eberly 1996), quantum mechanical image processing (Kneer & Law 1996) and a photon 'pistol' (Law & Kimble 1997).

We begin with a summary of the benefits attached to the most popular 'workhorse' of cavity QED, namely the Jaynes–Cummings system. These characteristics will be familiar to most workers in quantum optics.

(a) The Hamiltonian and the atom–field interaction are easily recognized as arising from first principles of QED.

(b) Exact solutions for time evolution of the interacting atom–field system are available without using perturbation theory or semiclassical decorrelations.

(c) Empirical linewidths and damping rates are not present, and there are no vanishing denominators at exact resonance.

(d) There are no runaway features of the time-dependences and no divergences.

Phil. Trans. R. Soc. Lond. A (1997) **355**, 2387–2391
Printed in Great Britain

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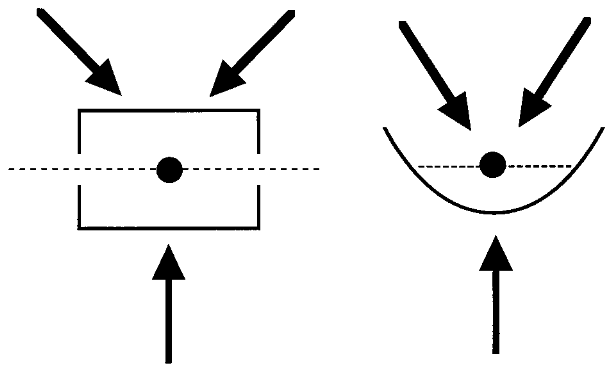

Figure 1. Three radiation fields are shown directed at an atom in a cavity or in a trap, which could be located in a cavity too. The cavity could support excitation of a fourth mode. The double-lambda model is compatible with three- or four-mode interactions.

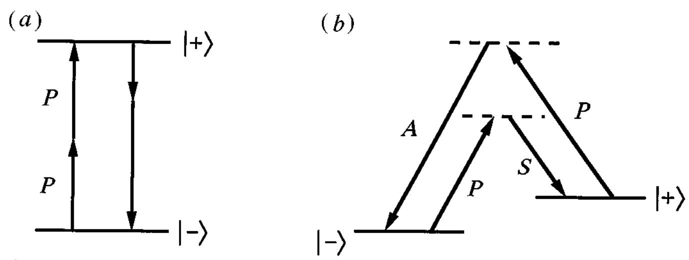

Figure 2. Two atom models that permit cyclic operation are shown. Both models can employ two 'pump' modes P and P′ that are different from each other, if necessary.

(e) Exact analytic solutions are easily written for both the eigenvalues and the dressed eigenstates of the fully interacting Hamiltonian.

With the Jaynes–Cummings system, there are many applications and dynamical consequences, some without classical analogues. One can think of the collapse and revival effects, production of entangled atom–field states, micromaser operation, novel methods to produce Fock states, squeezed states and sub-Poisson states, vacuum Rabi oscillations, etc. All in all, the model is a handy toy in which to examine fundamental aspects of both optical spectroscopy and laser action, with the major advantage that topics of current experimental interest can be treated in detail. Among these are several that have been already mentioned in this Discussion Meeting, particularly involving situations in which just one atom is sufficient to represent a very dense gas to the field, and in which just one photon can represent a very intense field to the atom.

It is not difficult to draw up an equally long unfulfilled 'wish list' of effects and processes that are well known in optical physics, but ones that don't fit into the Jaynes–Cummings framework. This list would include: (a) pump-probe spectroscopy; (b) nonlinear wave mixing, including down conversion; (c) three-mode correlations of the GHZ type; (d) two-photon lasing; and (e) optical pumping of laser action.

Two generic examples of atom–field models that are able to provide some or all of these effects and processes are shown in figure 2. Note that both employ radiation modes in such a way that there are two independent channels connecting the states $|+\rangle$ and $|-\rangle$.

It is possible for the atom in figure 2 to act catalytically as a mode-converter

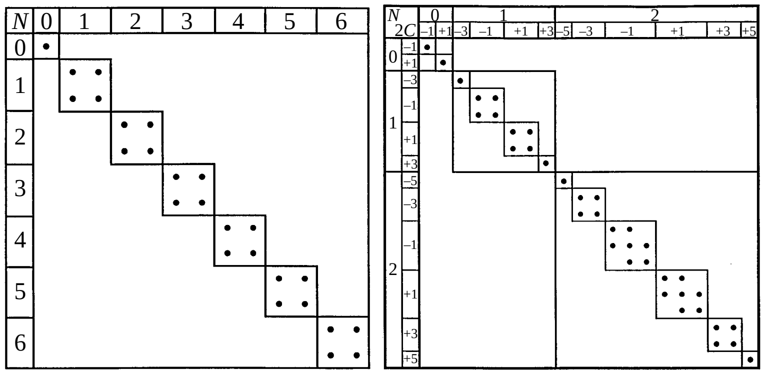

Figure 3. The form of the Hamiltonian matrices of the Jaynes–Cummings model (left) and the double-lambda model (right), taking account of all off-diagonal 'dressing' interactions included in equation (1.1).

in both cases by operating unidirectionally around a 'cycle'. Consider the double-lambda case, of direct concern below. The absorption of a pump photon (labelled P) and the accompanying emission of a Stokes (S) photon, followed by the absorption of a second pump photon and then the emission of an anti-Stokes (A) photon takes the atom from state $|-\rangle$ to state $|+\rangle$ and around to $|-\rangle$ again. The atom ends where it began, but the field inventory of photons is different. In the end there are two fewer pump photons and one additional each of Stokes and anti-Stokes photons. The atom has acted as a wave-mixing catalyst, and is ready to be used again. The Jaynes–Cummings system has only a single channel connecting its two states, so J–C cyclic operation of this kind is not possible.

For practical reasons, one must ask to what degree the more complex double-lambda remains sufficiently 'simple' and 'solvable.' To do this we compare the two systems' interaction Hamiltonians,

$$V_{\mathrm{J-C}} = g[\hat{a}\hat{\sigma}_{+-} + \mathrm{h.c.}], \quad V_{\Lambda-\Lambda} = g[(\hat{a}_{\mathrm{P}}\hat{a}_{\mathrm{S}}^{+} + \hat{a}_{\mathrm{P}}^{+}\hat{a}_{\mathrm{A}})\hat{\sigma}_{+-} + \mathrm{h.c.}]. \tag{1.1}$$

The matrix representation of the first is extremely simple, as given in figure 3*a*, since the number of 'excitations' is a constant (the number of photons plus the occupation number of the upper state cannot change under $V_{\mathrm{J-C}}$). The matrix representation of the second is more complicated because there is another good quantum number. The total number of photons is independent of the operator that is equivalent to the J–C 'excitation' number operator, so they must both be counted. When this is done systematically, the Λ–Λ Hamiltonian matrix appears as in figure 3*b*.

Of course, one knows why the Jaynes–Cummings is so simple and useful. A single generic quadratic equation diagonalizes all of the disconnected 2×2 matrices shown in figure 3*a*. The double lambda, by contrast, also consists of disconnected matrices, but matrices that get progressively bigger for larger numbers of photons in the modes, eventually reaching arbitrarily large size. Nevertheless, procedures have been identified (for both the three- and four-mode cases) which give the eigenvalues and eigenfunctions for this Hamiltonian exactly and analytically in closed form†. The key is the recognition of constants of the motion, including the 'conversion' operator given by $\hat{C} = \hat{n}_{\mathrm{A}} - \hat{n}_{\mathrm{S}} + \hat{\sigma}_{++}$, which is the equivalent of the J–C 'excitation' operator.

† The three-mode double-lambda model is diagonalized in Wang *et al.* (1992) and the four-mode model in Wang & Eberly (1993).

The equation to be solved is

$$V_{\Lambda-\Lambda}|N, 2C; \lambda\rangle = \lambda|N, 2C; \lambda\rangle, \tag{1.2}$$

where N and C are the quantum numbers that fix the total photon number and the conversion constant.

In the four-mode case, a straightforward application of angular momentum algebra provides the route to the solution. In the three-mode case, the solution follows from an expansion in bare atom–field states $|p, s, a; \pm\rangle$, in which the four indices count the numbers of photons in the pump, Stokes and anti-Stokes modes and the inversion. The cyclic action of the two-channel mode-mixing 'engine' underlying the three-mode Λ–Λ model converts the eigenvalue problem into a three-term recursion relation for the coefficients of the expansion. In the simplest case, when $C = -\frac{1}{2}$, we have

$$\begin{aligned}\lambda^2 c_n = (M-n)\sqrt{(2n+1)(2n+2)}c_{n+1} + [2n(2M-2n+1) + (M-n)]c_n \\ +(M-n+1)\sqrt{2n(2n+1)}c_{n-1}.\end{aligned} \tag{1.3}$$

For contrast, the equivalent Jaynes–Cummings recursion formula is simply (on resonance)

$$\lambda^2 c_n = g^2(n+1)c_n. \tag{1.4}$$

The Λ–Λ recursion relation implies a Riemann differential equation that has 24 possible hypergeometric solutions, but only one fits the requirement of providing a polynomial of the right order. The eigenvalue formula is remarkably simple,

$$\lambda^2 = 2m(m + |C|), \quad m = 0, 1, 2, \ldots. \tag{1.5}$$

Several optical effects compatible with the Λ–Λ system model were listed above, and nonlinear wave mixing is clearly realized in the cyclic operation of the double-lambda system. However, the most interesting applications, so far, of the Λ–Λ model were not on the list. Since the dynamical evolution is fully deterministic and unitary, two independent channels permit externally injected modes to deterministically control the remaining (quantized) mode. This is the basis for a photon 'engine' and the invention of a 'photon pistol.' Only the design for the photon engine (Law & Eberly 1996) has been published so far. It has been shown that a set of external pulses, that turn the pump-Stokes channel and the anti-Stokes-pump channel on and off alternately, can force the cavity mode to develop from vacuum as desired. After a finite total interaction time and a finite number of pulses, the cavity mode can be put into whatever superpositions of Fock states that a 'customer' might specify. We have described this as the basis for the first 'practical' non-classical photon state factory, practical in the sense that atom–field entanglement is avoided and there is no need to first construct an atomic state 'template' of the non-classical field state desired. The procedures for quantum image processing (Kneer & Law 1996) and the photon pistol (Law & Kimble 1997) are based on the same principles.

In summary, several interesting and unexpected applications of the strongly driven Λ–Λ system have recently been proposed. They make use of 'control' features provided by the two independent channels of the model. We expect that they are the first of a large number of applications, since both ion traps and optical microwave cavities are now providing practical experimental access to the domain of single-photon single-atom strong interactions.

C. K. Law and A. Rahman assisted the preparation of this article. The research reported here was supported by the US National Science Foundation.

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